

AY2017 Mathematical Analysis (1/8)

13:30-15:00, 4th October 2017

Functions and Graphs

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Objective of this course:

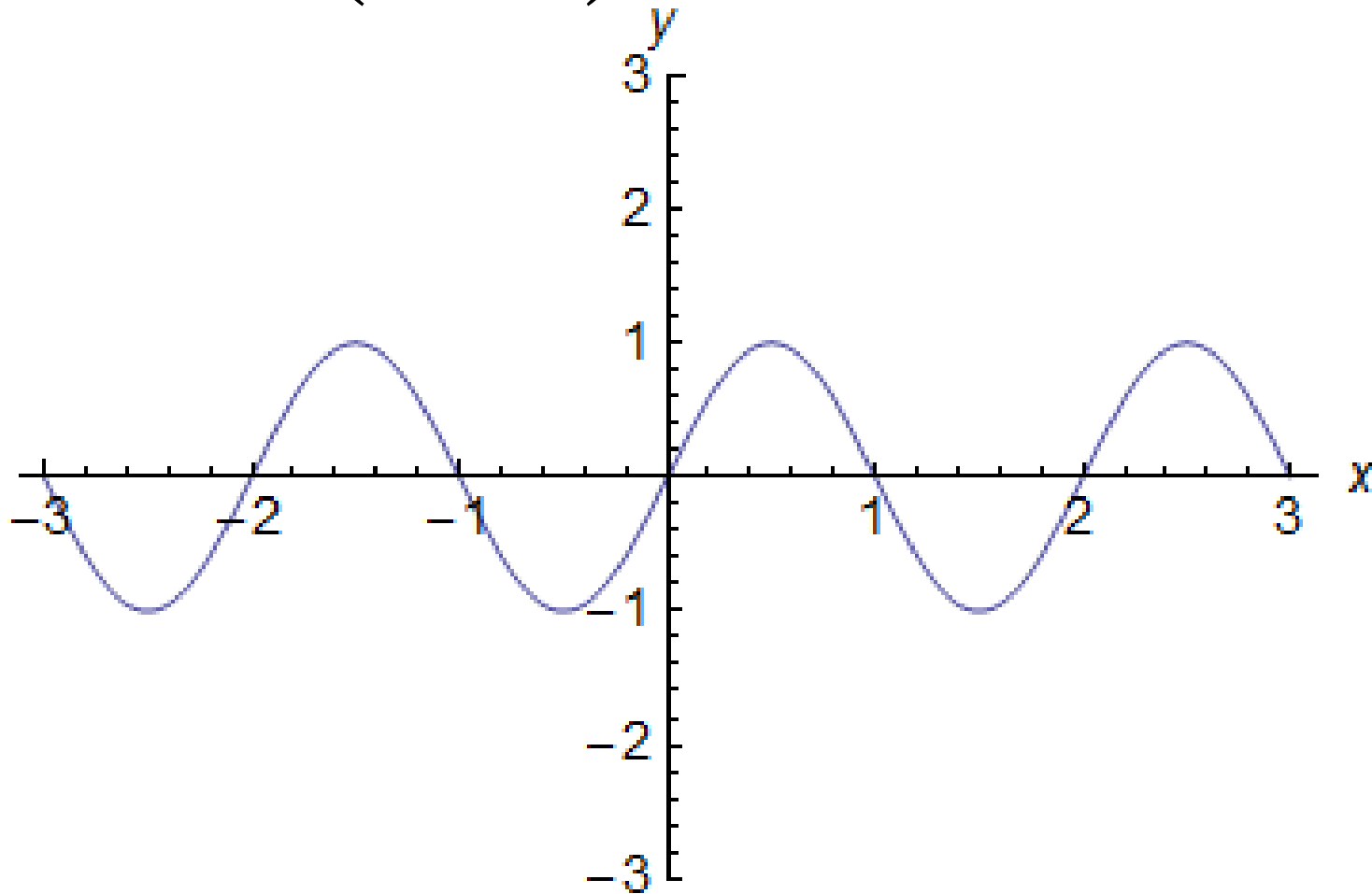
Quick review of basic mathematical techniques

Function and graph, derivation and integration, vector, matrix, complex number, error and statistics, differential equation

Only one rule in this course:

Calculate by yourself!

$$y = \sin(\pi x)$$



Important: shape, intersection points

Oscillation?

violin, drum, swing,

quartz watch,

earthquake,

molecule, crystal,

sound, light, electron

1. linear functions: $y = ax + b$

Let's plot graphs !

$$y = x$$

$$y = x + 3$$

$$y = 3x + 3$$

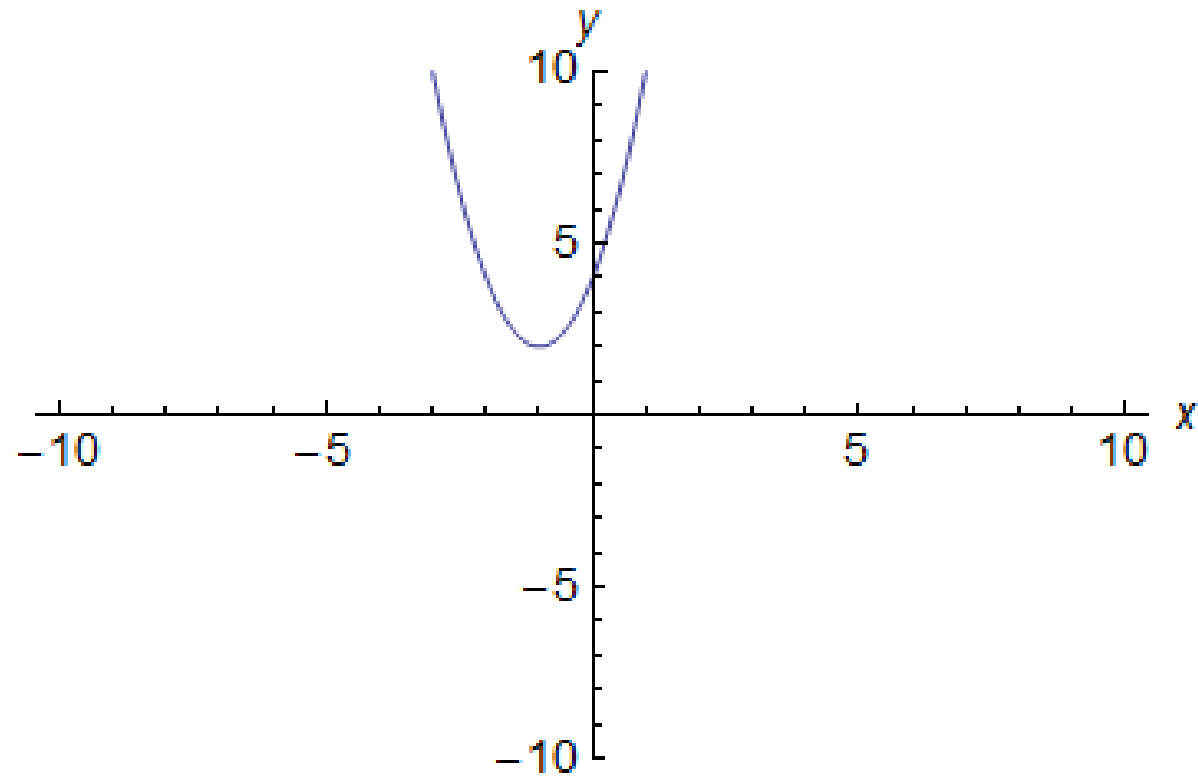
2. Quadratic functions:

$$y = ax^2 + bx + c$$

Plot graphs !

$$y = 2x^2 + 4x + 4$$

$$y = 2x^2 + 4x + 4$$



3. Exponential functions: $y = a^x$

$$y = 2^3 \times 2^2 = ?$$

$$y = \frac{2^2}{2^3} = ?$$

$$y = (2 \times 3)^2 = ?$$

$$y = 2^{-2} = ?$$

$$y = 4^{\frac{1}{2}} = ?$$

$$y = 2^0 = ?$$

3. Exponential functions: $y = a^x$

$$y = 2^3 \times 2^2 = \underline{32} \quad y = \frac{2^2}{2^3} = \frac{1}{\underline{2}}$$

$$y = (2 \times 3)^2 = \underline{36}$$

$$y = 2^{-2} = \frac{1}{\underline{4}}$$

$$y = 4^{\frac{1}{2}} = \underline{2}$$

$$y = 2^0 = \underline{1}$$

Advanced:
the base of natural logarithms

$$y = e^x$$

$$e = 2.7182818284 \dots$$

In your daily life,
“exponential” means “becoming faster and
faster, usually of a rate of increase”

4. logarithmic functions: $y = \log_a x$

For example, in case of $a=2$ and $x=8$,

Let's find y for $2^y = 8$

$$\log_2 1 = ?$$

$$\log_2 64 = ?$$

4. logarithmic functions: $y = \log_a x$

$$2^y = 8 \quad , \text{ therefore } y=3$$

$$\log_2 1 = \underline{0}$$

$$\log_2 64 = \underline{6}$$

Advanced: optical transmission and absorbance

$$-\log_{10} T = A$$

100% transmission

$$T = 1 \rightarrow A = ?$$

10% transmission

$$T = 0.1 \rightarrow A = ?$$

1% transmission

$$T = 0.01 \rightarrow A = ?$$

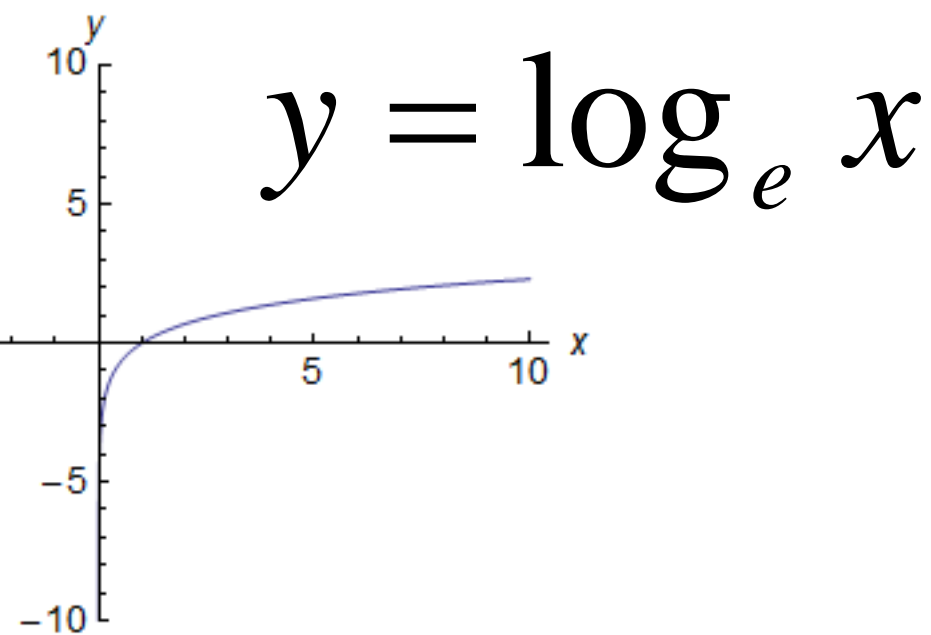
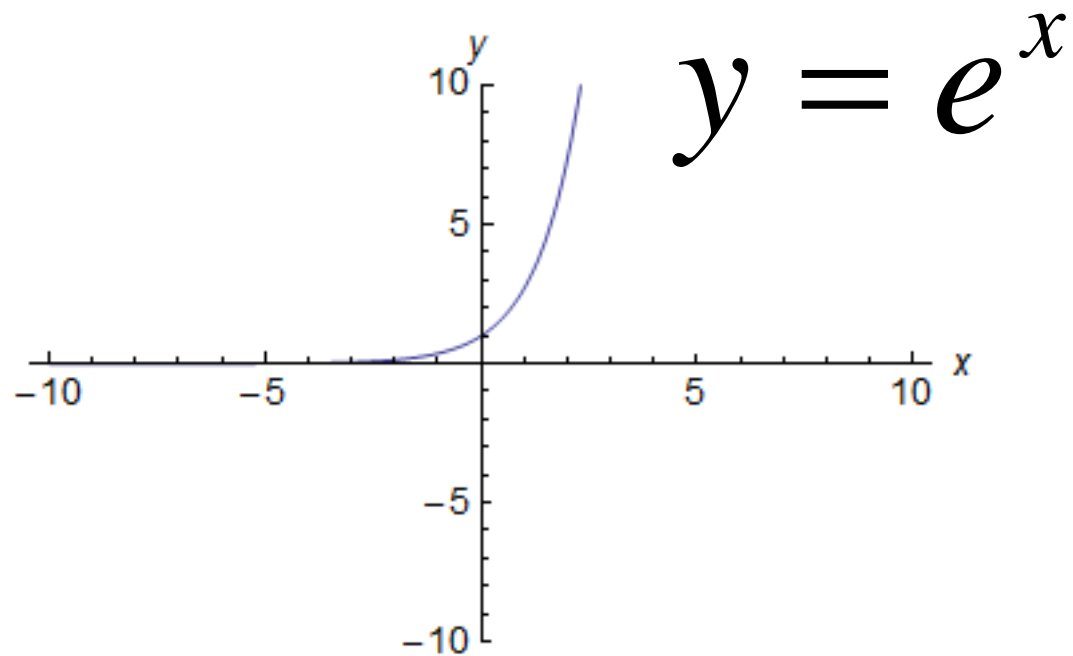
Advanced: optical transmission and absorbance

$$-\log_{10} T = A$$

$$T = 1 \rightarrow A = \underline{0}$$

$$T = 0.1 \rightarrow A = \underline{1}$$

$$T = 0.01 \rightarrow A = \underline{2}$$

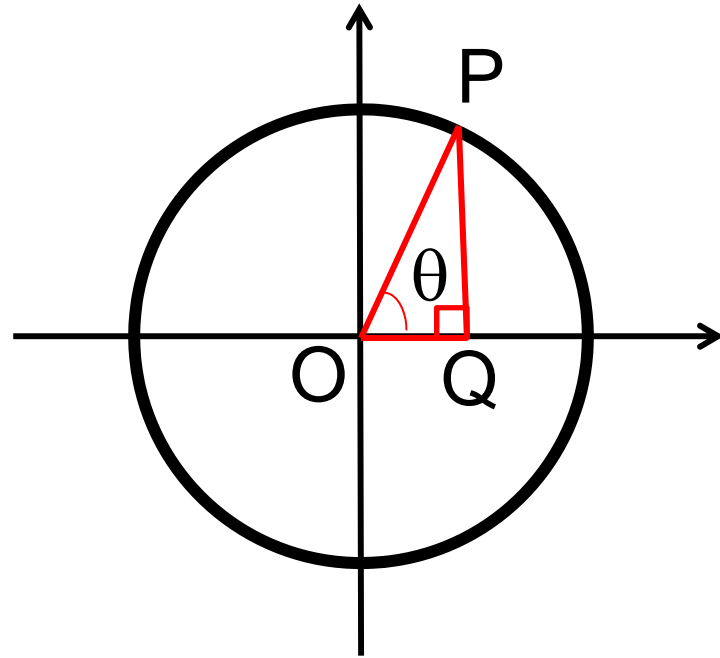


5. trigonometric functions

$$\sin \theta = \frac{PQ}{OP}$$

$$\cos \theta = \frac{OQ}{OP}$$

$$\tan \theta = \frac{PQ}{OQ}$$



$$\sin 0 = ?$$

$$\cos 0 = ?$$

$$\tan 0 = ?$$

$$\sin \frac{\pi}{2} = ?$$

$$\cos \frac{\pi}{2} = ?$$

$$\tan \frac{\pi}{2} = ?$$

$$\sin \pi = ?$$

$$\cos \pi = ?$$

$$\tan \pi = ?$$

$$\sin 0 = \underline{0} \quad \cos 0 = \underline{1} \quad \tan 0 = \underline{0}$$

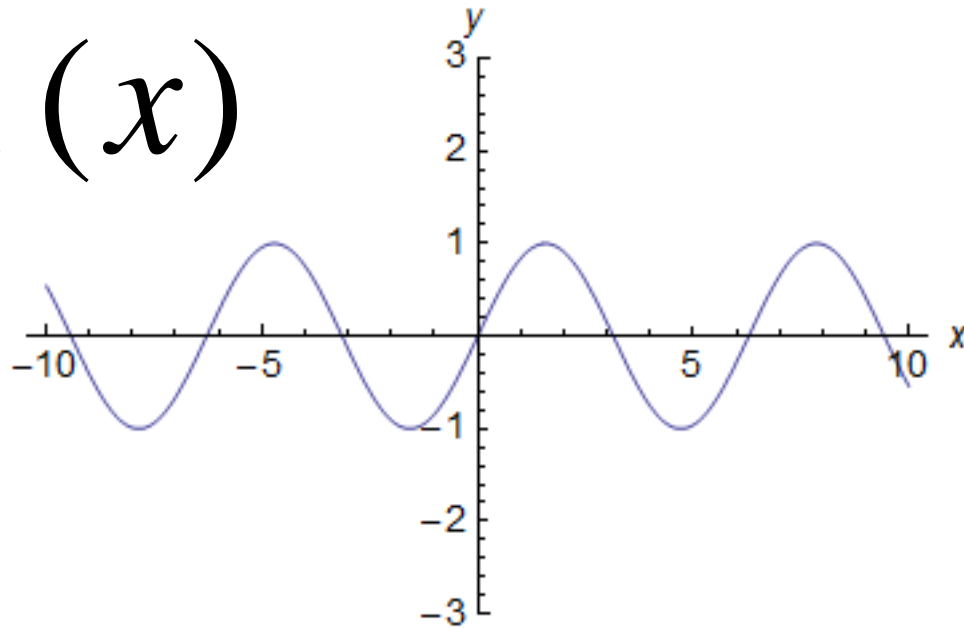
$$\sin \frac{\pi}{2} = \underline{1} \quad \cos \frac{\pi}{2} = \underline{0} \quad \tan \frac{\pi}{2} = \underline{\quad}$$

$$\sin \pi = \underline{0} \quad \cos \pi = \underline{-1} \quad \tan \pi = \underline{0}$$

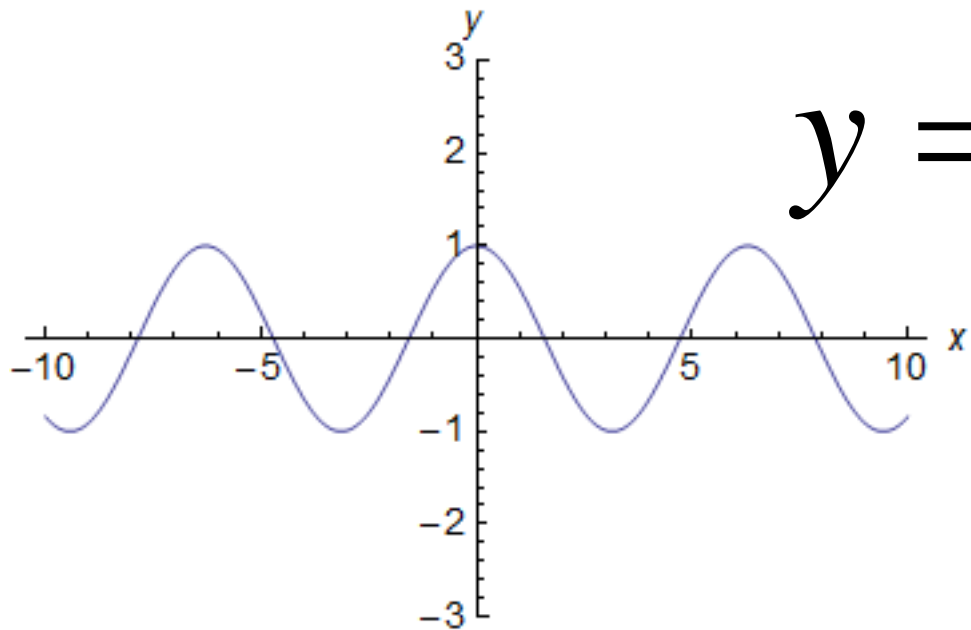
Let's plot graphs !

$$y = \sin x, y = \cos x$$

$$y = \sin(x)$$



$$y = \cos(x)$$



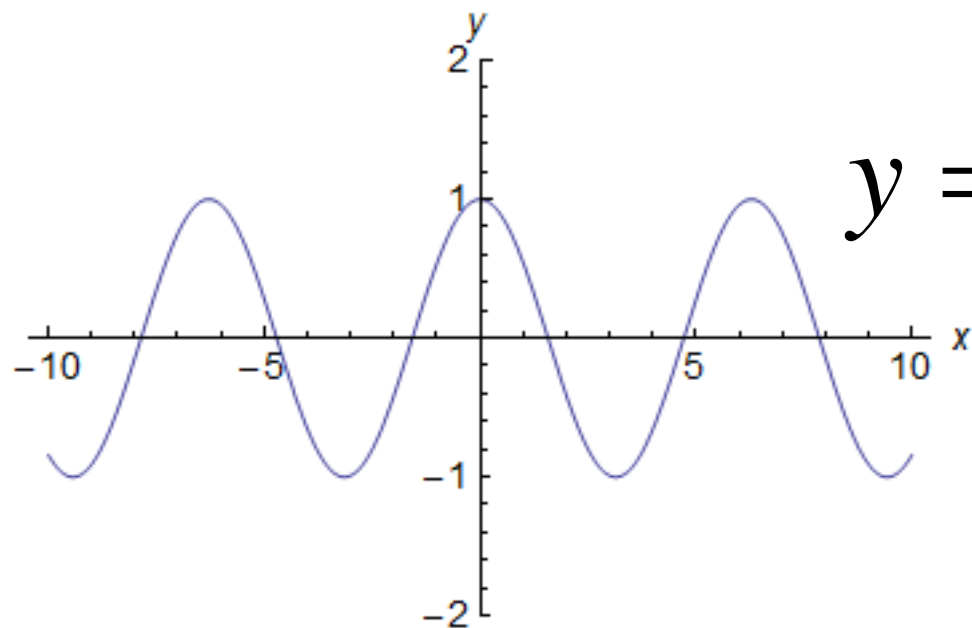
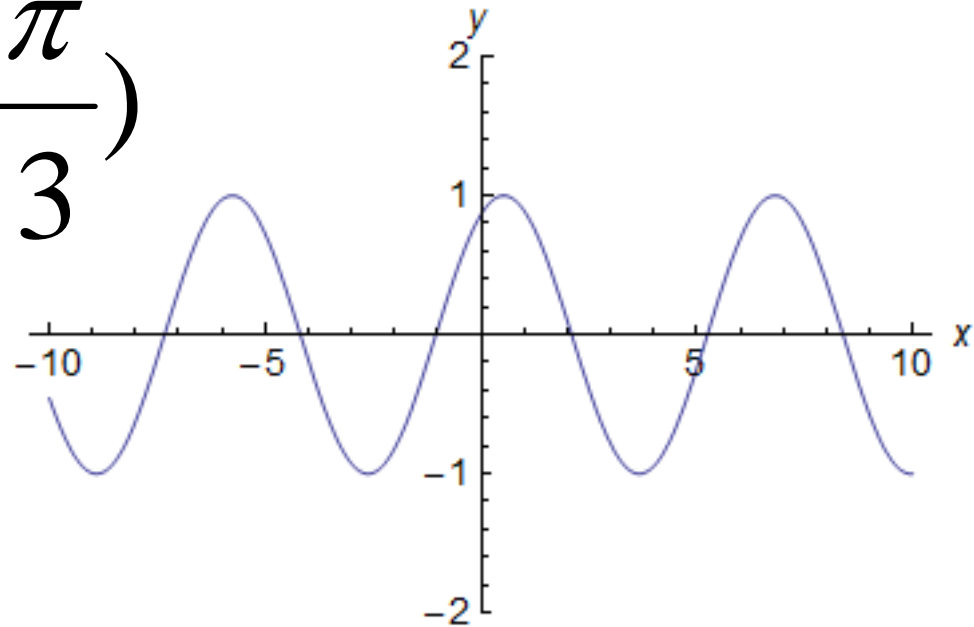
Plot graphs !

$$y = \sin x$$

$$y = \sin\left(x + \frac{\pi}{3}\right)$$

$$y = \sin\left(x + \frac{\pi}{2}\right)$$

$$y = \sin \left(x + \frac{\pi}{3} \right)$$



$$y = \sin \left(x + \frac{\pi}{2} \right)$$

Advanced:

◆ the Pythagorean theorem

$$\sin^2 \theta + \cos^2 \theta = 1$$

◆ the Euler's formula

$$\cos \theta + i \sin \theta = e^{i\theta}$$

$$\theta = \pi \rightarrow e^{i\pi} + 1 = 0$$