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Waseda Workshop on Partial Differential Equations 2019

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Venue: Large Conference Room, 1st Floor, 55N Bldg., Waseda University,
Nishi-Waseda Campus
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◆ Daoyuan Fang (Zhejiang Univ.) ◆

March 22, Friday　10:30-11:10

Almost global existence for the Kirchhoff-type equation with the periodic boundary conditions

This is the joint work with Zheng Han. In this paper, we prove an almost global existence result for the Kirchhoff-type equation on $\mathbb{T}^d$ with Cauchy data of small amplitude ϵ. We show a lower bound ϵ^{-N} for the existence time with any natural number N. This generalizes a preceding result concerning a lower bound ϵ^{-4}. The proof relies on the method of normal forms and induction. The difficulty is to find out the structure of the nonlinearity so that the process of normal forms can be performed up to any order.

◆ Shuichi Kawashima (Waseda Univ.) ◆

March 22, Friday　11:20-12:00

Nonlinear waves for a model system of hyperbolic balance laws

We consider a simple model system of hyperbolic balance laws:

$$\begin{aligned} &u_t + f(u)_x + q_x = 0, \\ &q_t + q + u_x = 0, \end{aligned} \tag{1}$$

where u and q are unknown functions of $x \in \mathbb{R}$ and $t > 0$, and the flux f is a smooth convex function of u. We can regarded u and q as the variables describing macroscopic and microscopic states in kinetic theory, respectively, and apply the Chapman-Enskog expansion to the system (1). Then, as the first order approximation (Euler level), we have $q = 0$ and a hyperbolic conservation law

$$u_t + f(u)_x = 0. \tag{2}$$

Also, as the second order approximation (Navier-Stokes level), we have $q = -u_x$ and a viscous conservation law

$$u_t + f(u)_x = u_{xx}. \tag{3}$$

We discuss the asymptotic stability of nonlinear waves for the system (1). We first observe that (1) admits a shock profile of the form $(u, q) = (\tilde{u}, \tilde{q})(x - st)$ which connects the constant states $(u_\pm, 0)$ with $u_- > u_+$, where s is a constant. This shock profile is an exact solution of the system (1). Under suitable assumptions we can prove the asymptotic stability of this shock profile $(\tilde{u}, \tilde{q})(x - st)$ as $t \to \infty$.

Next we consider a rarefaction wave of the form $(u, q) = (u^r, 0)(x/t)$, where $u^r(x/t)$ is the centered rarefaction wave (an exact solution) of the hyperbolic conservation law (2) which connects the constant states $u_\pm$ with $u_- < u_+$. We note that $(u, q) = (u^r, 0)(x/t)$ is not an exact solution of (1). Under suitable assumptions we can also prove the asymptotic stability of this rarefaction wave $(u, q) = (u^r, 0)(x/t)$ as $t \to \infty$.

◆ **Sijia Zhong (Southeast Univ.)** ◆

March 22, Friday 13:30-14:10

Well-posedness of supercritical wave equation on a compact Riemannian manifold with Dirichlet boundary condition

In this talk, we will study the global well-posedness of wave equation $\partial_t^2 u - \triangle u + |u|^{p-1}u = 0$ $1 < p < 5$ on a compact Riemannian manifold with Dirichlet boundary condition with some special kind of randomized initial data. We will show that this equation will be well-posed in $H^s(M) \times H^{s-1}(M)$ for $\frac{p-3}{p-1} < s < 1$ if $3 < p < 5$, and $0 < s < 1$ if $1 < p \leq 3$, and get some polynomial growth of the sobolev norm of the solution.

◆ **Chengbo Wang (Zhejiang Univ.)** ◆

March 22, Friday 14:20-15:00

The Strauss conjecture on negatively curved backgrounds

We provide a simple and geometric proof of small data global existence for the shifted wave equation on hyperbolic space involving nonlinearities of the form $\pm|u|^p$ or $\pm|u|^{p-1}u$. It is based on the weighted Strichartz estimates of Georgiev-Lindblad-Sogge (or Tataru) on Euclidean space. We also prove a small data existence theorem for variably curved backgrounds which extends earlier ones for the constant curvature case of Anker-Pierfelice, Anker-Pierfelice-Vallariono and Metcalfe-Taylor. It is based on the joint work with Yannick Sire and Chris Sogge.

◆ **Masashi Misawa (Kumamoto Univ.)** ◆

March 22, Friday 15:30-16:10

Finite singularity for the m-harmonic flow

The m-harmonic flow is the evolution of the m-harmonic maps between two smooth compact Riemannian manifolds, where the number m is the integer of the dimension of domain manifold. The m-harmonic maps are critical points of the m-energy for maps

between two smooth compact Riemannian manifolds.
The m-harmonic map is an extension of the two-dimensional harmonic map to the higher dimension. In such case that the integral exponent is equal to the domain dimension, we have a scaling transformation, under which the local energy and the equation are invariant. We know that a weak solution of the m-harmonic map is regular, because of the scaling invariance and the small energy regularity theorem. The m-harmonic flow is the negative directed gradient flow of the m-energy and has a fundamental application to the m-harmonic maps. In this talk, we consider the regularity of the m-harmonic flow and in particular we study an estimation of size of the singular set. Our main theorem is the global existence of a weak solution to the m-harmonic flow with large energy data, which is regular except at most finitely many points.

◆ Makoto Nakamura (Yamagata Univ.) ◆

March 22, Friday 16:20-17:00

Remarks on a semilinear diffusion equation in homogeneous and isotropic spaces

We consider the Cauchy problem (P') given by

$$(P')\begin{cases}\dfrac{\partial u}{\partial s}(s,x)-\Delta u(s,x)=A(s)f(u)(s,x) & \text{for } (s,x)\in[0,S_0)\times\mathbb{R}^n,\\ u(0,\cdot)=u_0(\cdot), \end{cases}$$

where we have put

$$A(s):=(1-2Hs)^{n(p-1)/4-1} \tag{0.1}$$

and

$$S_0:=\begin{cases}\frac{1}{2H} & \text{if } H>0,\\ \infty & \text{if } H<0.\end{cases} \tag{0.2}$$

In this talk, we consider the existence of the solutions for small data, and the asymptotic behaviors of them. The existence part is given as follows for some function space $X(R)$.

Theorem 1. *Let $H\in\mathbb{R}, n\ge 1, 1<p<\infty$.*

(1)(Small global solutions for $H>0$) Let $H>0$. If $H^{-1/(p-1)}\|u_0\|_{L^\infty}$ is sufficiently small, then there exists a unique global solution u for (P') in $X(R)$ for some $R>0$.

(2)(Small global solutions for $H<0$) Let $H<0$. If $M^{1/(p-1)}\|u_0\|_{L^1\cap L^\infty}$ is sufficiently small, then there exists a unique global solution u for (P') in $X(R)$ for some $R>0$. Here, we have put

$$M:=\begin{cases}(-2H)^{n(p-1)/4-1} & \textit{if } p\ge 1+\frac{4}{n},\ H<-\frac{1}{2},\\ (-2H)^{n(p-1)/4-1} & \textit{if } p<1+\frac{4}{n},\ -\frac{1}{2}\le H<0,\\ 1 & \textit{if } p\ge 1+\frac{4}{n},\ -\frac{1}{2}\le H<0,\\ 1 & \textit{if } p<1+\frac{4}{n},\ H<-\frac{1}{2}.\end{cases}$$

◆ **Hiroshi Takeda (Fukuoka Inst. Tech.)** ◆
March 22, Friday 17:10-17:50

Asymptotic expansion of solutions for wave equations with structural damping and its application

Wave equations with damping term arise from many context including mathematical physics. In this talk, we firstly study the Cauchy problem of wave equations with structural damping term;

$$(1)\qquad \begin{cases} \partial_t^2 u - \Delta u + \nu(-\Delta)^\sigma \partial_t u = 0, & t > 0, \quad x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \quad \partial_t u(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases}$$

where $u : (0,\infty) \times \mathbb{R}^n \to \mathbb{R}$, $0 \leq \sigma \leq 1$ and $\nu > 0$. Our main interest is the asymptotic profiles of the solutions to (1) as $t \to \infty$ and they are summarized as follows:

Theorem 1. Let $n \geq 2$ and $k_0 \geq 0$. Assume that $(u_0, u_1) \in (H^{k_0+1} \cap L^1) \times (H^{k_0} \cap L^1)$. Then, the asymptotic profiles of solutions to (1) is classified into 6 patterns depending on ν, σ, a and b.

We note that our main contribution to Theorem 1 is in the case $\frac{1}{2} < \sigma < 1$.

As a consequence of Theorem 1, we can conclude the lower bounds of the solutions.

Corollary 2. If $\displaystyle\int_{\mathbb{R}^n} u_1(x)dx \neq 0$, there exists $C > 0$ such that

$$C^{-1} t^{-\gamma_{\sigma,k} - \ell} \leq \| \partial_t^\ell \nabla_x^k u(t) \|_2 \leq C t^{-\gamma_{\sigma,k} - \ell}, \quad 0 < \sigma \leq \tfrac{1}{2},$$
$$C^{-1} \sqrt{\log(t+1)} \leq \| u(t) \|_2 \leq C \sqrt{\log(t+1)}, \quad \tfrac{1}{2} < \sigma \leq 1 \text{ and } n = 2,$$
$$C^{-1} t^{-\gamma_{\sigma,k} - \frac{\ell}{2\sigma}} \leq \| \partial_t^\ell \nabla_x^k u(t) \|_2 \leq C t^{-\gamma_{\sigma,k} - \frac{\ell}{2\sigma}}, \quad \tfrac{1}{2} \leq \sigma \leq 1 \text{ and otherwise}$$

for $t \gg 1$, $\ell \in \{0,1\}$ and $k \in [0, k_0+1]$ with $k + \ell \leq k_0 + 1$, where

$$\gamma_{\sigma,k} := \begin{cases} \frac{n}{4(1-\sigma)} - \frac{\sigma}{1-\sigma} + \frac{k}{2(1-\sigma)}, & 0 \leq \sigma \leq \frac{1}{2}, \\ \frac{n}{4\sigma} - \frac{1}{2\sigma} + \frac{k}{2\sigma}, & \frac{1}{2} \leq \sigma \leq 1. \end{cases}$$

Next, we apply Theorem 1 to the elastic waves with structural damping:

$$(2)\qquad \begin{cases} \partial_t^2 u - a^2 \Delta u - (b^2 - a^2) \nabla(\mathrm{div}\ u) + B \partial_t u = 0, \quad t > 0, \quad x \in \mathbb{R}^n, \\ u(0,x) = f(x), \quad \partial_t u(0,x) = g(x), \quad x \in \mathbb{R}^n, \end{cases}$$

where $u = (u_1, u_2, \cdots, u_n)$, $B = (-\Delta)^\sigma$ or $(-a^2\Delta - (b^2 - a^2)\nabla \mathrm{div})^\sigma$, $f(x) = (f_1, f_2, \cdots, f_n)$ and $g(x) = (g_1, g_2, \cdots, g_n)$, which are recently studied by [1], [2] and [3]. We will deduce the asymptotic profiles of solutions to (2) is given by 8 patterns for the suitable initial data.

References

[1] Ikehata, R., Charão, R.C. and da Luz, C. R., J. Evol. Equ. 14 (2014) 197-210.
[2] Reissig, M., Math. Methods Appl. Sci. 39 (2016), 4618-4628.
[3] Wu, M., Chai, S. and Li, S., J. Math. Anal. Appl. 452 (2017), 361-377.

◆ **Ting Zhang (Zhejiang Univ.)** ◆
March 23, Saturday 10:30-11:10

Local and global existence of pathwise solution for the stochastic Boussinesq equations with multiplicative noises

In this talk, we consider the stochastic Boussinesq equations in $\mathbb{T}^d$ with the nonlinear multiplicative noises. At first, we establish the local existence of pathwise solutions. Then, we establish the global existence of pathwise solution when the noises are non-degenerate, which show that the linear multiplicative noises would provide a regularizing effect: the global existence of solution occurs with high probability if the initial data are sufficiently small, or if the noise coefficients are sufficiently large. (Based on the work with Lihuai Du)

◆ **Jiang Xu (Nanjing Univ. of Aeronautics and Astronautics)** ◆
March 23, Saturday 11:20-12:00

The optimal time-decay for the compressible Navier-Stokes equations in the critical L^p framework

The global existence issue for the isentropic compressible Navier-Stokes equations in the critical regularity framework has been addressed by R. Danchin more than fifteen years ago. However, whether (optimal) time-decay rates could be shown in general critical spaces and any high dimensions has remained an open question. In this talk, we survey recent results not only in the L^2 critical framework but also in the more general L^p critical framework, which were exactly as firstly observed by A. Matsumura and T. Nishida in the case $p = 2$ and $d = 3$, for solutions with high Sobolev regularity.

References

[1] R. Danchin and J. Xu: Optimal time-decay estimates for the compressible Navier-Stokes equations in the critical L^p framework, *Arch. Rational Mech. Anal.*, **224**, 53-90 (2017).

[2] Z. Xin and J. Xu: Optimal decay for the compressible Navier-Stokes equations without additional smallness assumptions, *arXiv:1812.11714v1* (2019).

[3] J. Xu: A low-frequency assumption for optimal time-decay estimates to the compressible Navier-Stokes equations, *Comm. Math. Phys.* (2019).

◆ **Guixiang Xu (Beijing Normal Univ.)** ◆
March 23, Saturday 13:30-14:10

Recent developments about the stability of the solitary waves for the (generalized) derivative Schrödinger equation

We will survey some recent developments about the stability of the solitary waves for the (generalized) derivative Schrödinger equation. We firstly show the existence/nonexistence of traveling waves for NLS with derivative (DNLS) by the structure analysis, ODE argument and the variational argument. Secondly, we consider the orbital stability of the solitary waves for the derivative Schrödinger equation in the energy space. Last, we will show the orbital instability of the solitary waves for the generalized derivative Schrödinger equation in the degenerate case. They are joint works with Prof. Changxing Miao and Xingdong Tang.

◆ **Ruizhao Zi (Central China Normal Univ.)** ◆
March 23, Saturday 14:20-15:00

Convergence to equilibrium for the solution of the full compressible Navier-Stokes equations

We study the convergence to equilibrium for the full compressible Navier-Stokes equations on the torus T3. Under the conditions that both the density ρ and the temperature θ possess uniform in time positive lower and upper bounds, it is shown that global regular solutions converge to equilibrium with exponential rate. We improve the previous result obtained by Villani in [Mem. Amer. Math. Soc., 202(2009), no. 950] on two levels: weaker conditions on solutions and faster decay rates. This is a joint work with Prof. Zhifei Zhang.

◆ **Masayuki Hayashi (Waseda Univ.)** ◆
March 23, Saturday 15:30-16:10

Potential well theory for the derivative nonlinear Schrödinger equation

We study the derivative nonlinear Schrödinger equation by variational approach. The main aim of this talk is to investigate global existence from the viewpoints of the solitons. By using variational arguments inspired from Payne–Sattinger's classical work we clarify the connection between mass condition for global existence and potential well generated by the solitons. We see that the effect of the momentum plays an essential role in the argument.

◆ **Erika Ushikoshi (Yokohama National Univ.)** ◆
March 23, Saturday 16:20-17:00

Hadamard variational formula for the multiple eigenvalues of the Stokes equations with friction slip boundary conditions

We consider the perturbation problem for the eigenvalue of the Stokes equations in a bounded domain $\Omega \subset \mathbb{R}^3$ with a smooth boundary $\partial\Omega$. More precisely, for any $\varepsilon \geq 0$ and for any $\rho \in C^\infty(\partial\Omega)$, we define a perturbed domain by Ω_ε, whose boundary is

$$\partial\Omega_\varepsilon := \{x + \varepsilon\rho(y)\nu_y \ ; \ y \in \partial\Omega\}.$$

Under such a smooth perturbation, we consider the eigenvalue problem of the Stokes equations with the slip friction boundary condition;

$$\begin{cases} \Delta\boldsymbol{\Phi}_\varepsilon + \lambda(\varepsilon)\boldsymbol{\Phi}_\varepsilon = \nabla q_\varepsilon & \text{in } \Omega_\varepsilon, \\ \operatorname{div} \boldsymbol{\Phi}_\varepsilon = 0 & \text{in } \Omega_\varepsilon, \\ (2e(\boldsymbol{\Phi}_\varepsilon)\nu_x + \kappa\boldsymbol{\Phi}_\varepsilon)_\tau = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\boldsymbol{\Phi}_\varepsilon = (\Phi^1_\varepsilon(x), \Phi^2_\varepsilon(x), \Phi^3_\varepsilon(x))$ and $q = q(x)$ is unknown velocity fields and pressurre at $x \in \Omega_\varepsilon$, respectively. Furthermore, κ is a positive friction constant, and $e(\boldsymbol{u}) = \{e_{ij}(\boldsymbol{u})\}_{1\leq i,j\leq 3}$ is the strain tensor defined by

$$e_{ij}(\boldsymbol{u}) := \frac{1}{2}\left(\frac{\partial u^i}{\partial y^j} + \frac{\partial u^j}{\partial y^i}\right), \quad i, j = 1, 2, 3.$$

It is a known fact that the spectral set of (1) consists of the descrete sequence of positive real numbers. Then, if we arrange them with counting multiplicity, we see that

$$0 < \lambda_1(\varepsilon) \leq \lambda_2(\varepsilon) \leq \cdots \leq \lambda_k(\varepsilon) \leq \cdots \to \infty,$$

where the k-th eigenvalue is denoted by $\lambda_k(\varepsilon)$. In this talk, we consider the domain dependency of the multiple eigenvalue $\lambda(\varepsilon)$. Indeed, we shall establish the Hadamard variational formula for the multiple eigenbalue of (1). This is a joint work with Professor Shuichi Jimbo in Hokkaido University.