EFFECTS OF HETEROGENEITY OF ROCK MASS ON SURFACE TILT

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1. INTRODUCTION

Evaluation of groundwater flow is required in fields of such as disposal of high-level nuclear wastes and storage of CO_2. For this purpose, a technique for monitoring tilt on the surface with a high resolution by using tiltmeters attracts attention, and a technique for evaluating groundwater flow at great depth from tilt data measured on the surface was proposed by Vasco et al. (1998) [1]. Furthermore, Nakatani et al. [2], Narikawa et al. [3] and Matsuki et al. [4] revised this method to evaluate groundwater flow more accurately by an inverse analysis of tilt data. However, these conventional methods are applicable only for a homogeneous half-space of a poroelastic medium. Thus, the aim of this study is to clarity the effects of mechanical heterogeneity of rock, surface topography and fault on surface tilts. In this study, 3D finite element analysis was performed for three simple models and a model with the geological structure of the Tono district, Gifu prefecture.

2. METHOD

2.1 Conventional Method

The conventional method is based on the relation between the tilt $t_i(x)$ at point x on the surface of a semi-infinite poroelastic body and fluid volume change per unit rock volume $\Delta v(s)$ at a point s in a region V, as shown in Fig. 1. The tilt $t_i(x)$ can be expressed as

$$t_i(x) = B \int_V \Delta v(s) T_i(x,s) dV, \qquad (1)$$

where $t_i(x)$ is the tilt in the X_i-direction, B is Skempton coefficient of the rock and $T_i(x,s)$ is given by

$$T_i(x,s) = \frac{v_u + 1}{\pi} \frac{(x_i - s_i)(x_3 - s_3)}{S^5}, \qquad (2)$$

where v_u is the undrained Poisson's ratio of the rock and S is given by

$$S = \sqrt{(x_1 - s_1)^2 + (x_2 - s_2)^2 + (x_3 - s_3)^2}. \qquad (3)$$

In inversion, $\Delta v(s)$ is determined by the least squares method for observation equations obtained from Eq. (1), which is conditioned by the sum of the second deivatives of Δv with a weighting coefficient.

2.2 Finite element method

When incompressible fluid volume change per unit rock volume Δv occurs, the constitutive law of a
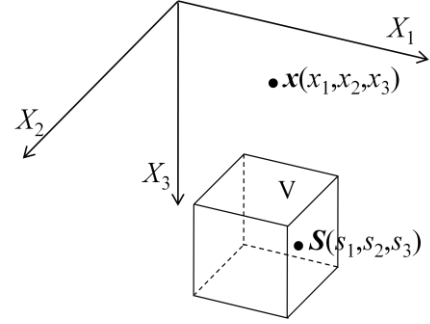

Fig. 1 Relation between a point on the surface (x) and a region where water flow occurs (V).

poroelestic medium is given by

$$\sigma = D\varepsilon - BK_u \Delta v j, \qquad (4)$$

where σ is stress, ε is strain and D is given by

$$D = \begin{bmatrix} 2\mu + \lambda_u & \lambda_u & \lambda_u & 0 & 0 & 0 \\ \lambda_u & 2\mu + \lambda_u & \lambda_u & 0 & 0 & 0 \\ \lambda_u & \lambda_u & 2\mu + \lambda_u & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \qquad (5)$$

and j is given by

$$j = (1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0)^t, \qquad (6)$$

where undrained Lamé constant λ_u and undrained volumetric bulk modulus K_u are expressed by:

$$\lambda_u = \frac{2v_u}{1 - 2v_u} \mu, \quad K_u = \frac{2(1 + v_u)}{3(1 - 2v_u)} \mu, \qquad (7)$$

where μ is rigidity and expressed with Young's modulus E and Poisson's ratio v as given by

$$\mu = \frac{E}{2(1 + v)}. \qquad (8)$$

Thus, Eq. (4) is determined by Young's modulus E, drained Poisson's ratio v, undrained Poisson's ratio v_u and Skempton coefficient B. The displacements for all nodes are calculated in FEM and that for an arbitrary point is interpolated from these at nodes. Thus, the surface tilts in the x and y direction at an arbitrary point are given by

$$t_x = \frac{\partial w}{\partial x}, \ t_y = \frac{\partial w}{\partial y} \qquad (9)$$

where w is the vertical displacement.

3. MODEL ANALYSIS
3.1 Method

In this section, I determine the effects of mechanical heterogeneity of rock, surface topography and fault on surface tilts by forward analysis for simple models, to compare the tilt obtained for each simple models with that for a homogeneous half-space model. Furthermore, a subsurface volumetric change is estimated by the conventional inversion method from the estimated tilt data and compared with the given values.

In the simple model analysis, the volume change region (V region) was assumed to be ellipsoid and defined by

$$\frac{x'^2}{x_m^2} + \frac{y'^2}{y_m^2} + \frac{z'^2}{z_m^2} = 1, \tag{10}$$

where (x', y', z') is the local coordinates of the arbitrary points in the ellipsoid and x_m, y_m, z_m are sizes in each direction ($x_m = 200$ m, $y_m = 200$ m, $z_m = 40$ m). The distribution of Δv is given by

$$\Delta v = a_0 \left(1 - \sqrt{\frac{x'^2}{x_m^2} + \frac{y'^2}{y_m^2} + \frac{z'^2}{z_m^2}} \right), \tag{11}$$

where

$$a_0 = \frac{3 V_0}{\pi x_m y_m z_m}, \tag{12}$$

where total volume change $V_0 = 100$ m^3 and $a_0 = 5.97$ e-5. Note that I assumed an increase in groundwater volume.

All models in this section are a rectangular parallelepiped with the size of 2000 m * 2000 m * 1000 m.

3.2 MODELS AND RESULTS
3.2.1 Heterogeneous models (Model 1)

Model 1 is composed of two layers. Young' moduli of the lower and upper rocks are 50.10 GPa and 10.02 GPa, respectively, and undrained Poisson's ratio is the same (0.35). The thickness of the upper layer is 160 m. Young's modulus and undrained Poisson's ratio of the homogeneous half-space model to be compared with Model 1 are the same as those of the lower layer. The depth of center of V region was assumed to be 120 m, 160 m and 200 m.

In Fig. 2, the surface tilt in the x direction obtained for the heterogeneous rock is compared with that obtained for the homogeneous rock, for the depth of V region of 200 m. This figure indicates that the tilt obtained for the heterogeneous rock is greater than that for the

homogeneous rock. Fig. 3 shows the vertical displacements at the top and bottom planes of the rectangular parallelepiped V region. For the heterogeneous rock, the displacement at the plane of the bottom of the rectangular parallelepiped is smaller and that at the plane of the top is greater than that of the homogeneous rock. This indicates that the deformation at the bottom of V region is suppressed by the lower hard rock, which in turn makes the displacement at the top of V region greater in the heterogeneous rock. Accordingly, the surface tilt of the heterogeneos model becomes greater than that of the homonegeous rock.

3.2.2 Model with the uneven surface (Models 2-1 and 2-2)

Model 2-1 has a mountian and Model 2-2 has a valley. Two models are homogeneous with Young's modulus of 50.10 GPa, undrained Poisson's ratio of 0.35 and Skempton coefficient of 0.9. Fig. 4 shows the contour plots of tilt x in unit of 10^{-6} rad for the homogeneous
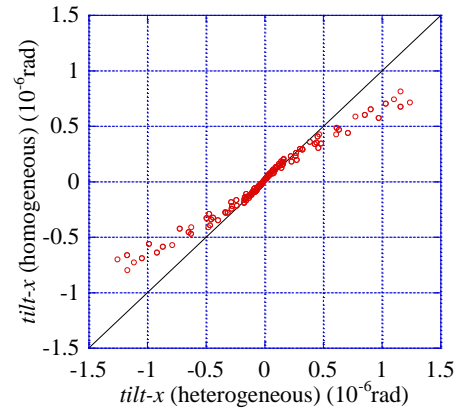

Fig. 2 Comparison of the surface tilt t_x obtained by the heterogeneous model with that by the heterogeneous model for $z_{V0} = 200$ m
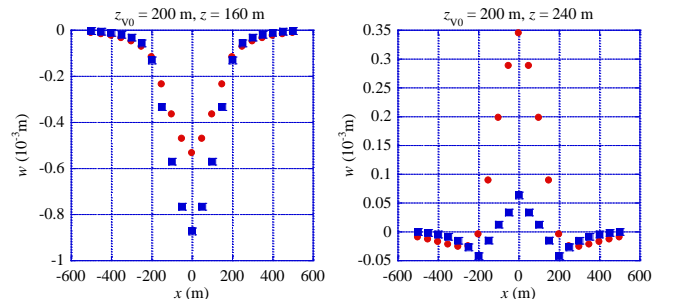

Fig. 3 Distribution of vertical displacement (w) at y = 0 m and at z = 160 m and 240 m for z_{V0} = 200 m. Circles and squares indicate the results for the homogeneous and heterogeneous models, respectively.

half-space, Model 2-1 (mountain) and Model 2-2 (valley). The square, the dashed straight line and the dashed-dotted line indicate the center of V region, the mountain ridge and the bottom line of the valley, respectively. Compared to the homogeneous half-space, the tilt obtained for the mountain model is smaller and that for the valley model is greater. As shown in Eq. (1), surface tilt is determined by the distance between V region and the observation station. For the mountain model, the distance from V region is greater than that for the homogeneous half-space, which reduces the surface tilt. In contrast, the tilt obtained for the valley model increases since the distance from V region is smaller than that for the homogeneous half-space.

3.2.3 Model with fault (Model 3)

In this study, the joint element proposed by Goodman [5] was used as a mechanical model for fault. In the joint element, two flat plates are coupled with springs in the normal and shear directions. The stiffnesses of each spring are given by

$$k_n = \frac{E_f}{t}, \; k_s = \frac{E_f}{2(1 + v_f)t}, \tag{13}$$

where E_f, v_f and t are Young's modulus, Pisson's ratio and thickness of a fault, respectively and $E_f = 0.501$ GPa, $v_f = 0.40$, $t = 10$ m. Model 3 has a single vertical fault in a homogeneous rock with Young's modulus of 50.1
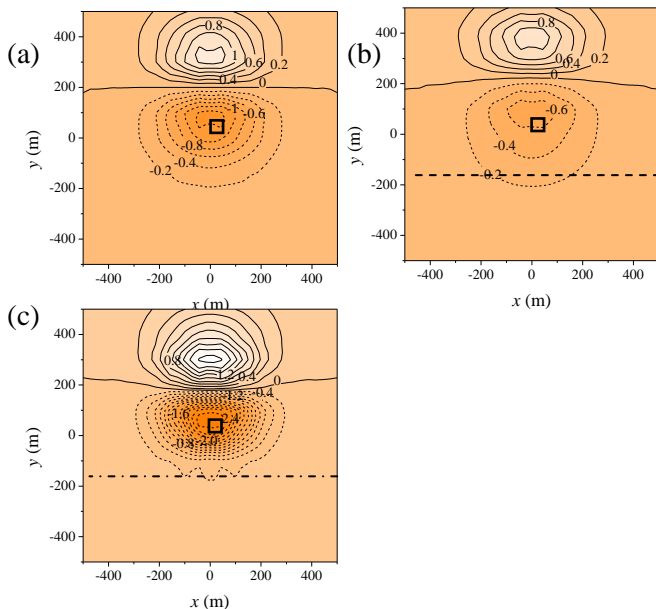

(a)
(b)
(c)

GPa, undrained Poisson ratio of 0.35 and Skempton coefficient of 0.9. The fault is located at $y = 0$ m with a strike along the x axis.

Fig. 5 shows the contour plots of tilt y obtained for the homogeneous half-space and Model 3. The result shows that the surface tilt decreases beyond the fault, because the fault absorbed the volumetric strain caused by the subsurface groundwater flow.

4. Tono model

The ultimate goal of this study is to develop a new inverse method using FEM, to evaluate the hydrological structure for a field, especially for the Tono district. However, all of the mechanical properties of rock mass in the Tono district are not known. Therefore through this analysis, I hope I can determine ranges of the mechanical properties in the Tono district. However, I write only a part of the results in the abstract.

The field is the site of the Mizunami underground research laboratory in the Tono district, Japan. In this site, the Main and Ventilation shafts are being excavated with drainage of groundwater. Four tiltmeters have been installed on the surface and are called ME02, ME03, ME04 and ME05 from the north. In the period from October 28 to December 10, 2005, excavation of shaft was stopped due to contamination of halogens in groundwater and the shafts was submerged. Narikawa [3] applied the conventional inverse method to the site and estimated the hydrological structure for this period, which showed that water volume change mainly occurred in the ellipsoidal region with the center at 150 m south the Main shaft. Thus, in this study, I assumed this ellipsoid as V region for the forward analysis. The size of V region is $x_m = 300$ m, $y_m = 150$ m, $z_m = 125$ m, where x and y axes direct the north and east, respectively, and the z axis directs the downward vertical direction. I classified the Tono district into three geological layers

(a)
(b)

Fig. 4 Surface tilt (t_x) obtained for (a) the homogeneous half-space model, (b) Model 2-1 (mountain) and (c) Model 2-2 (valley).
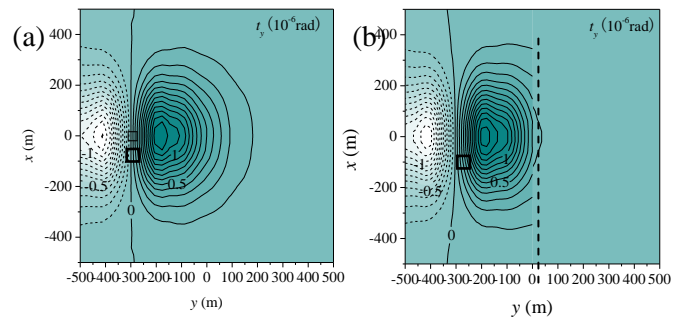
Fig. 5 Surface tilt (t_y) obtained for (a) the homogeneous half-space and (b) Model 3.

from the surface: sedimentary rock, upper fracture zone in granite and granite from the surface. In this analysis, I determined the standard values of the mechanical properties for each layer, based on Kaneko(2006) : for sedimentary rock, Young' modulus E_s = 2.00 GPa, drained Poisson's ration v_s = 0.40, and Skempton coefficient B = 0.9; for granite, E_g = 20.0 GPa, v_g = 0.30 and B = 0.9. For the upper fracture zone, Young's modulus increases linearly with depth from E_s to E_g, and v = 0.30, B = 0.9. In this study, I analyzed surface tilt for various combinations of mechanical properties for each layer and the shape and position of V region, to investigate the effect of the heterogeneity and the shape and position of V region.

Fig. 6 shows the surface tilt obtained for (a) the homogeneous and (b) heterogeneous Tono models. The major axis of V region directs the north for the homogeneous Tono model and the NNW for the heterogeneous Tono model. Fig. 6(b) is the result closest to the observed values in this study. The positions of the shafts and tiltmeters are drawn in an appropriate scaling and the tilt data are plotted as variations from the position of each tiltmeter. Solid and open symbols indicate the estimated observed tilts, respectively. The result for the homogeneous Tono model shows that the estimated tilts for ME04 and ME05 differ only slightly from the observed values directionally and quantitatively, while the east component of tilts for ME02 and ME03 are less than the observed values. Thus, the estimated tilts are not consistent with the observed data. On the other hand, for the heterogeneous Tono model, the east components of tilts for ME02 and ME03 are larger than that for the homogeneous Tono model. Thus, the results show that the major axis of V region directs the NNW and the best values of the mechanical properties that reproduce the observed values most closely are E_s = 2.00 GPa, v_s = 0.10, v_{us} = 0.15, B = 0.9, E_g = 20.0 GPa, v_g = 0.10, v_{ug} = 0.15, and B = 0.9.

5. CONCLUSIONS

In this study, first I evaluated the effect of heterogeneity of rock mass, surface topography and fault on surface tilt through simple model analyses. The result shows that surface tilt is affected by the heterogeneity of rock mass, surface topology and a fault. Second, I analyzed the model with geological structure of the Tono district to investigate ranges of the mechanical properties of each geological layer. The
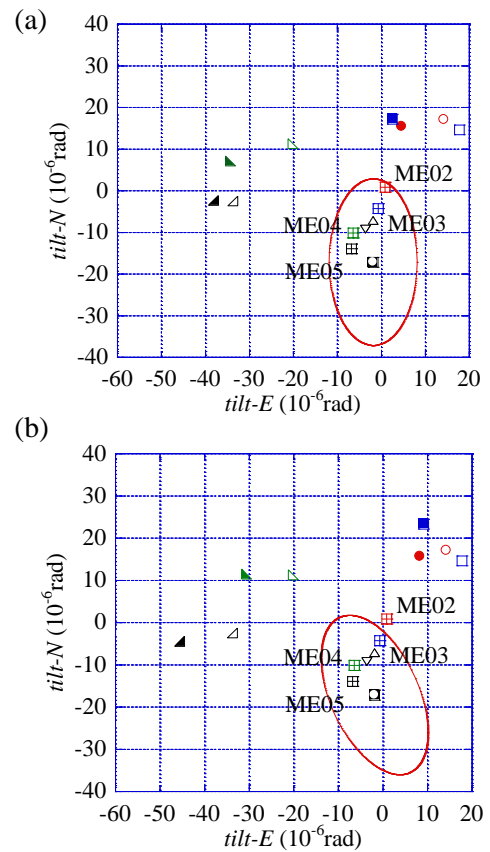

Fig. 6 Surface tilt obtained for (a) the homogeneous and (b) the heterogeneous Tono models.

result shows that the best values of the mechanical properties which reproduce the observed values most closely are E_s = 2.00 GPa, v_s = 0.10, v_{us} = 0.15, B = 0.9, E_g = 20.0 GPa, v_g = 0.10, v_{ug} = 0.15, and B = 0.9.

I hope this study would be great help for developing a new method to evaluate groundwater flow, in future.

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