A FUNDAMENTAL STUDY ON EVALUATION OF UNDERGROUND WATER FLOW BASED ON TILT ON THE SURFACE

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1. INTRODUCTION

A technique for evaluating underground fluid flow is required now in fields of such as geothermal energy development and disposal of nuclear wastes. On the other hand, with civilian use of the tiltmeter which has been developed as a military sensor, a technique for monitoring tilt on the surface with a high resolution by using the tiltmeter attracts attention, since a technique for evaluating water flow at great depth from tilt data measured on the surface was proposed by Vasco et al. (1998)[1]. However, this method has some problems. Hence, first, revising the method proposed by Vasco et al., I developed a technique for evaluating ground water flow more accurately by an inverse analysis of tilt data. Second, to estimate hydrologic structure from tilt data measured on the surface during excavation of two shafts, I applied to this technique to the Tono district, Japan.

2. CONVENTIONAL METHOD AND THE PROBLEMS

The method proposed by Vasco et al. is based on the relation between the tilt $t_i(\boldsymbol{x})$ at a point $\boldsymbol{x}$ on the surface of a semi-infinite poroelastic body and fluid volume change per a unit rock volume $\Delta v(\boldsymbol{s})$ at a point $\boldsymbol{s}$ in a region V, as shown in Fig. 1. The tilt $t_i(\boldsymbol{x})$ can be expressed as

$$t_i(\boldsymbol{x}) = \frac{1}{\rho_0}\int_{V} B\Delta v(\boldsymbol{s}) T_i(\boldsymbol{x},\boldsymbol{s})dV, \quad (1)$$

where $t_i(\boldsymbol{x})$ is the tilt in the X_i-direction, B is Skempton coefficient of the rock, ρ_0 is the density of the fluid and $T_i(\boldsymbol{x},\boldsymbol{s})$ is given by

$$T_i(\boldsymbol{x},\boldsymbol{s}) = \frac{\nu+1}{\pi}\frac{(x_i - s_i)(x_3 - s_3)}{S^5}, \quad (2)$$

where ν is Poisson's ratio of the rock and S is given by

$$S = \sqrt{(x_1 - s_1)^2 + (x_2 - s_2)^2 + (x_3 - s_3)^2}. \quad (3)$$

After dividing the region V into K cells, Vasco et al. rewrote Eq. (1) in a matrix form as

$$\boldsymbol{t} = \boldsymbol{G}\boldsymbol{v}, \quad (4)$$

where the vector $\boldsymbol{t}$ contains M tilt components, $\boldsymbol{v}$ is a vector of the fluid volume change of the cells and G is a $2M \times K$ matrix. To find the fluid volume change $\boldsymbol{v}$, Vasco et al. applied the least squares method to Eq. (4). However, it is usually difficult to get a unique solution because there are much more numbers of fluid volume change to be determined than observation data to use. Therefore, Vasco et al. added a weighed sum of the square of the first derivative of Δv to the error derived from Eq. (4), and minimized the total error to get a unique solution. However, in the inverse analysis method suggested by Vasco et al., there are some problems as shown below:

1) Because fluid volume change is constant in each cell, many cells are necessary to evaluate it more precisely. However, the precision in the evaluation becomes worse as the number of cells increase since unknowns increase.
2) Fluid volume change Δv is not zero at the boundary of the region V. Accordingly, the region where the fluid flow occurs is not clearly defined.
3) Vasco et al. used the first derivative of Δv as a condition of constraint in their inverse analysis, but this may distort a real distribution of the fluid flow.

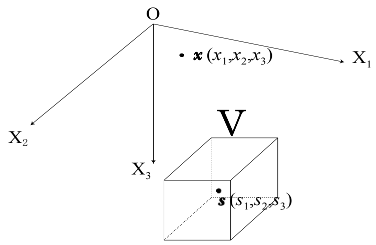

Fig. 1 Relation between a point on the surface ($\boldsymbol{x}$) and a region V where water flow occurs.

3. MODEL ANALYSIS

I revised the inverse analysis method proposed by Vasco et al. as follows:

1) Fluid volume change Δv and Skempton coefficient B at an arbitrary point in a cell are interpolated linearly or quadratically from the values at the nodal points of each cell.
2) Fluid volume change Δv is set to zero on the boundary of the region V to evaluate underground fluid volume change more precisely.
3) I used the sum of the square of the second derivative of Δv as a condition of constraint in the least squares method.

I called the new inverse method using linear interpolation Inverse-1 and the method using quadratic interpolation Inverse-2.

Model analyses were performed using the new inverse methods. In this study, fluid volume change Δv was evaluated for two kinds of water injection test in the region V. These are the cases that fluid volume change occurs slowly and rapidly from the injection point. I called the former Model 1 and the latter Model 2. First, tilt data at nine points of these cases were calculated by forward analyses. Then, to confirm the effectiveness of the new inverse methods, fluid volume change Δv were evaluated using the tilt data to compare the results with given values of Δv.

Fig. 2 (a) shows Model 1. The region V is a rectangular solid of 320 m×280 m×240 m. This region was divided into 8×8×8 elements. In addition, an injection point 0' lying at a depth of 500 m from the surface is set on the center of the region V. Rock mass is assumed to be granite (Skempton coefficient B = 0.9, Poisson's ratio ν = 0.25). When total fluid volume change V_0 is injected from the point 0', fluid volume change is distributed over an ellipsoid. The ellipsoid is 160 m in the X'-direction (x_m), 140 m in the Y'-direction (y_m) and 120 m in the Z'-direction (z_m). Fluid volume change Δv is given by

$$\Delta v = a_0(1-r^*), \quad (5)$$

where a_0 is a coefficient determined from V_0 and r^* is a variable ranging from 0 to 1 given by

$$r^* = \sqrt{\frac{x'^2}{x_m^2}+\frac{y'^2}{y_m^2}+\frac{z'^2}{z_m^2}}. \quad (6)$$

Fig. 3 shows the results obtained by inverse analysis for Model 1. The relation between the increase of fluid volume and the distance from the injection point is shown. Circles indicate the theoretical values calculated by Eq. (5) at each nodal point. Squares are the results estimated by Inverse-1 and diamonds are results by the method of Vasco et al.. When the results by Inverse-1 are compared with the values from Eq. (5), there are some errors. These errors were due to the large number (729) of points for fluid volume change compared to the numbers (2×9) of tilt data. However, Fig. 3 shows that the results by Inverse-1 are much more accurate than those by the method proposed by Vasco et al..

Fig. 2 (b) shows Model 2. The region V whose center is the injection point 0' is a cube with 320 m on a side and is divided into 8×8×8 elements. The injection point lies at a depth of 500 m from the ground level. The theoretical solution of fluid volume change Δv in the coordinate (r, θ, ϕ) in the case that fluid flows into an infinite body at constant flow rate q is given by

$$\Delta v(r,t) = \frac{q}{4\pi c r}\int_0^t erfc\left(\frac{r}{2\sqrt{cz}}\right)dz, \quad (7)$$

where r is the distance from the injection point, c *is* the diffusion coefficient, t is time from the start of injection and $erfc(x)$ is the complementary error function. Rock mass was assumed to be granite (c = 9.25 m^2/s, B = 0.9). In addition, q was assumed to be 1.16×10^{-3} m^3/s which is equivalent to the case that water of $100 m^3$ are injected for 24 hours.

Fig. 4 shows the results estimated by Inverse-2 for Model 2. The increase in fluid volume is shown as a function of the distance from the injection point. The curve in the figure shows the theoretical value obtained by Eq. (7). Circles indicate the results estimated by Inverse-2 and squares indicate those by the method of Vasco et al.. As a result, large errors were found when the results by Inverse-2 are compared with the theoretical solution. It is considered that this is because Inverse-2 cannot express rapid volume change in the vicinity of the injection point as given by Eq. (7). In addition, these large errors were considered again to be produced by the much greater number (2673) of points for fluid volume change than number (2×9) of tilt data.

Therefore, in this study, to reduce the number of parameters to be determined, I expressed fluid volume change Δv with multinomials as

$$\Delta v = \sum_{k=1}^{n} a_k(1-r^*)^n, \quad (8)$$

where r^* is given by Eq. (6) and n is the number of the term. By substituting Eq. (8) into Eq. (1), the parameters to be determined is reduced only to coefficients a_k. I called this inverse method Inverse-3. Fluid volume change Δv for Model 2 were evaluated using Inverse-3. Diamonds in Fig. 4 show the results estimated by inverse-3 when n = 5 for which the results were the closest to the theoretical values by Eq. (7).

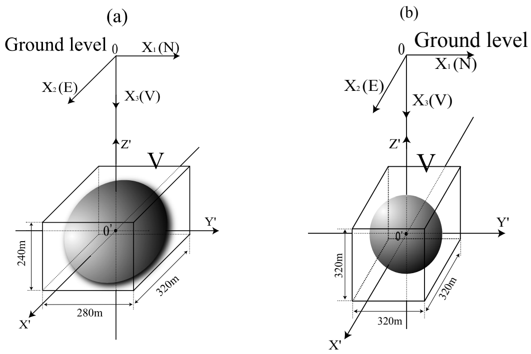

Fig. 2 (a) Model 1 and (b) Model 2.

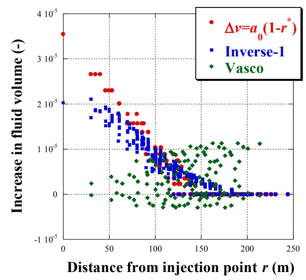

Fig. 3 Results by Inverse-1 for Model 1.

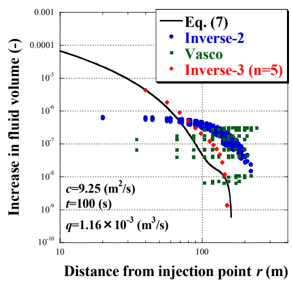

Fig. 4 Results by Inverse-2 and Inverse-3 for Model 2.

4. INFLUENCE OF THE SIZE OF REGION OF FLUID VOLUME CHANGE ON ESTIMATION

In the previous model analyses, the actual size of the region of fluid volume change was always given. Therefore, the correct size of the region was used when the inverse analyses were conducted. However, it is usually difficult to accurately estimate the region V of fluid volume change in fields. Hence, it is necessary to evaluate the influence of the size of the region on inverse analysis when the region of fluid volume change is different from the actual region. The model used here is the same as that in Fig. 2 (a) Model 1. However, the fluid Volume change Δv was given by

$$\Delta v = 0.025(1-r^*) + 0.015(1-r^*)^2 + 0.01(1-r^*)^3, \quad (9)$$

where r^* is given by Eq. (6).

To evaluate the influence of the size of the region of fluid volume change on the results, following inverse analyses were conducted:

1. The case that only the horizontal size of region of fluid volume change is changed.
2. The case that only the vertical size is changed.

Figs. 5 and 6 show the error in Δv which were obtained by the two inverse analyses described above and were normalized by the maximum value. Fig. 5 shows the results for the case that the size in the horizontal direction of the region V (x_m) was changed and Fig. 6 shows those for the case that the size of the vertical direction (z_m) was changed. The results show that the influence of the size of the region is relatively small for both Inverse-1 and Inverse-2 even if the region is larger than the real value. In addition, accuracy doesn't fall down for these methods unless an extremely small region is used. On the other hand, the results by Inverse-3 were strongly influenced by the size of the region of fluid volume change even if the difference from the actual size is slight.

Thus, how to apply the new inverse methods to fields is summarized as below:

1) You should use Inverse-1 or Inverse-2 unless the size of the region of fluid volume change in the field can be estimated with a high accuracy.

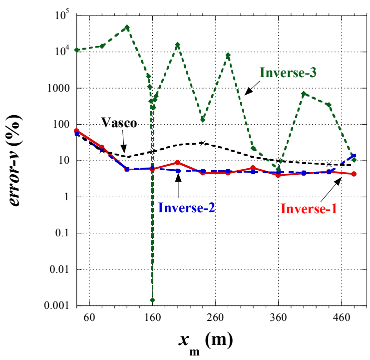

Fig. 5 The error in fluid volume change when the inversion region is changed only for the X'-direction.

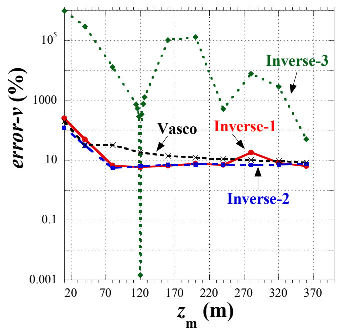

Fig. 6 The error in fluid volume change when the inversion region is changed only for the Z'-direction.

2) When the new inverse methods are applied to fields, you should assume a large size for the region of water flow, and then reduce the size.

5. APPLICATION TO A FIELD

The field is the site of Mizunami underground research center in the Tono district, Japan. In the site, drainage of ground water which flowed from surrounding rock masses into main and ventilation shafts with the excavation, has been conducted until September, 2005. Therefore, by using tilt data measured with tiltmeters on the surface, I estimated hydrological structure around the site.

Fig. 7 shows the positions of the shafts and tiltmeters around the site of the research center [2). Two faults (IF_SB1_004 and IF_SB1_005 faults with a strike in the direction of north-northwest - south-southeast) were estimated to exist by previous research. Four tiltmeters have been installed on the surface and are called ME02, ME03, ME04 and ME05 from the north. I evaluated the distribution of Δv for the period from 12:00 on April 21 to 23:00 until September 30, 2005. Fig. 8 shows the tilt data in the north and east directions measured by each tiltmeter. I evaluated fluid volume change by Inverse-2 during the period. Fig. 9 shows the field model used in the analysis. This is a rectangular model of 1120 m×1120 m×160 m. The center 0' of the region of fluid volume change V lies at a depth of 160 m which the main shaft reached at the end of the period.

Fig. 10 shows the result of ground water flow distributions estimated by Inverse-2. The fluid volume change distribution in the horizontal plane of a depth 160 m of main shaft is shown as a contour map. In this study, decrease in fluid volume was taken as positive. So, the area of Δv > 0 indicates the region where fluid volume decreases, while the area of Δv < 0 indicates the region where the volume increases. The region where fluid volume decrease lies almost parallel to the two faults. Therefore, it can be said that the ground water flowed into the shafts came from the region along these two faults. In addition, Fig. 11 shows the contour map of the elevation of the boundary between sedimentary rocks and granite estimated by previous research [3]. This boundary surface descends from the site to the southeastern direction, and the site is located on a valley of granite. Because the direction and region of this valley almost coincide with those of the region where fluid volume decreases, it can be said that ground water flow came from the thick sedimentary rock formation on the valley of granite.

6. CONCLUSIONS

In this study, I firstly revised the method proposed by Vasco et al. and developed a new technique for accurately evaluating ground water flow by an inverse analysis using tilt data.

Model analyses were conducted using the inverse methods I developed. In the case that fluid volume change occurs slowly in the region V (Model 1), Inverse-1 can evaluate the fluid volume change accurately. However, in the case that fluid volume change occurs rapidly near an injection point (Model 2), even Inverse-2 can not produce good results. Hence, to reduce the parameters to be determined, I expressed fluid volume change Δv in multinomials (Inverse-3) and applied it to Model 2. As a result, the results by Inverse-3 agreed to theoretical values.

I evaluated the influence of the size of the region of fluid volume change on inverse analysis results and suggested the following application ways of the inverse methods.

1) You should use Inverse-1 or Inverse-2 unless the size of the region of fluid volume change in fields can be estimated with a high accuracy.
2) When the new inverse methods are applied to fields, you should use a large region in the beginning and then decrease the region.

Then, I estimated hydrological structure of the Tono district from tilt data measured during shafts excavation. As a result, it was shown that ground water which flowed into the two shafts mainly came from sedimentary rock lying on the valley of the granite and also from the region along the two faults existing near the shafts.

In future, the reliability of this method can be improved by developing the evaluation method that considers geologic structures around the field.

REFERENCES

1) Vasco, D. W., Karasaki, K. and Myer, L., (1998). Monitoring of fluid injection and soil consolidation using surface tilt measurements, *J. Geotech. Geoenv. Eng.*, pp29-37.

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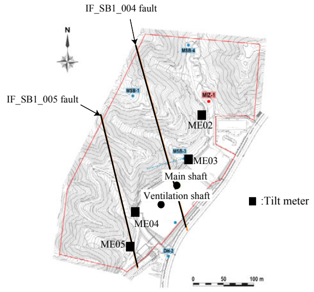

Fig. 7 Positions of shafts and tiltmeters.

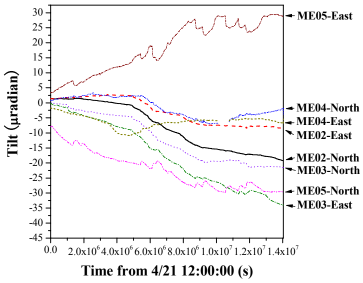

Fig. 8 Tilt data after removing background noises.

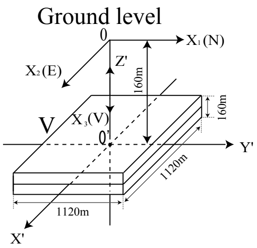

Fig. 9 Assumed region of water flow.

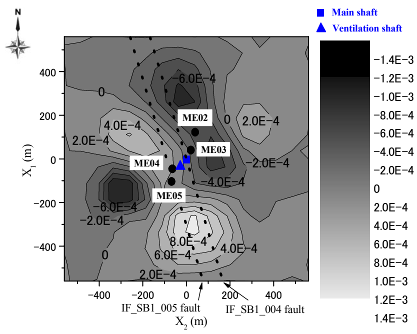

Fig. 10 Fluid volume change at elevation of 160 m estimated by Inverse-2 for the whole period.

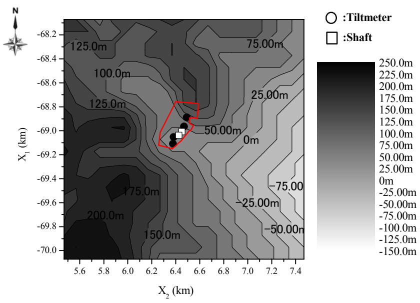

Fig. 11 Contour map of the sedimentary rocks – Toki granite boundaries.